

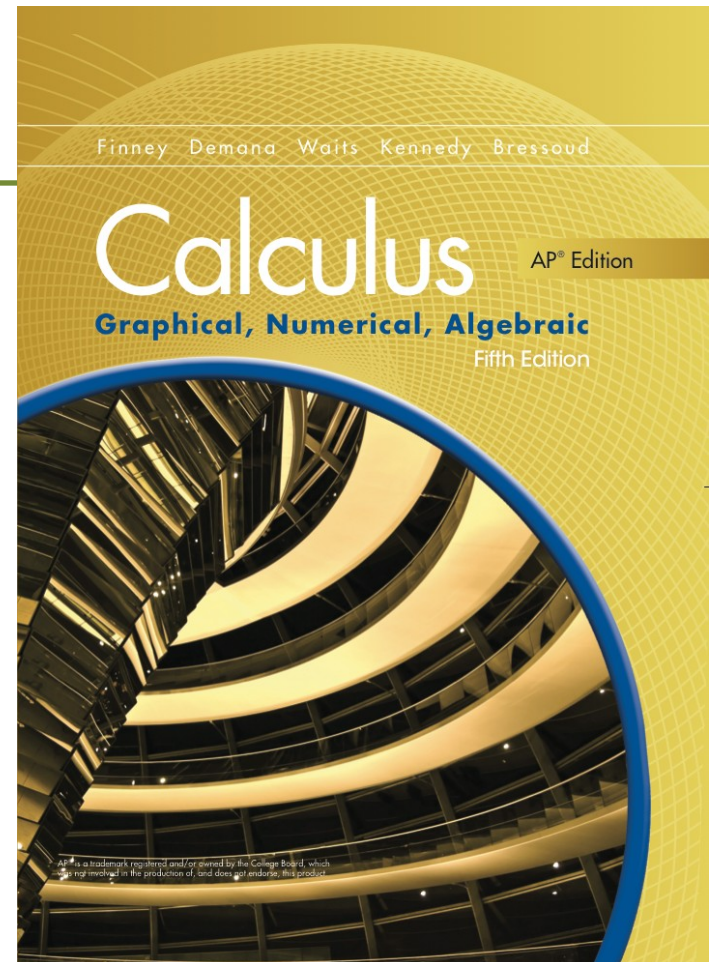
Chapter 1

Prerequisites for Calculus



Section 1.5

Functions and Logarithms



Quick Review

In Exercises 1- 4, let $f(x) = \sqrt[3]{x-1}$, $g(x) = x^2 + 1$, and evaluate the expression.

1. $(f \circ g)(1)$

2. $(g \circ f)(-7)$

3. $(f \circ g)(x)$

4. $(g \circ f)(x)$

Quick Review

In Exercises 5 and 6, choose parametric equations and a parameter interval to represent the function on the interval specified.

5. $y = \frac{1}{x-1}, x \geq 2$

6. $y = x, x < -3$

Quick Review

In Exercises 7- 10, find the points of intersection of the two curves. Round your answers to 2 decimal places.

7. $y=2x-3$, $y=5$

8. $y=-3x+5$, $y=-3$

9. (a) $y=2^x$, $y=3$

(b) $y=2^x$, $y=-1$

10. (a) $y=e^x$, $y=4$

(b) $y=e^x$, $y=-1$

Quick Review

In Exercises 1- 4, let $f(x) = \sqrt[3]{x-1}$, $g(x) = x^2 + 1$, and evaluate the expression.

1. $(f \circ g)(1) = 1$

2. $(g \circ f)(-7) = 5$

3. $(f \circ g)(x) = x^{\frac{2}{3}}$

4. $(g \circ f)(x) = (x-1)^{\frac{2}{3}} + 1$

Quick Review

In Exercises 5 and 6, choose parametric equations and a parameter interval to represent the function on the interval specified.

(other answers are possible)

5. $y = \frac{1}{x-1}, x \geq 2$

6. $y = x, x < -3$

$x = t, y = \frac{1}{t-1}, t \geq 2$

$x = t, y = t, t < -3$

Quick Review

In Exercises 7- 10, find the points of intersection of the two curves. Round your answers to 2 decimal places.

7. $y=2x-3$, $y=5$ (4, 5)

8. $y=-3x+5$, $y=-3$ $\left(\frac{8}{3}, -3\right)$

9. (a) $y=2^x$, $y=3$ (1.58, 3)

(b) $y=2^x$, $y=-1$ none

10. (a) $y=e^x$, $y=4$ (-1.39, 4)

(b) $y=e^x$, $y=-1$ none

What you'll learn about

- One-to-one functions and the horizontal line test
- Finding inverse functions graphically and algebraically
- Base a logarithm functions
- Properties of logarithms
- Changing bases
- Using logarithms to solve exponential equations algebraically

...and why

Logarithmic functions are used in many applications including finding time in investment problems.

One-to-One Functions

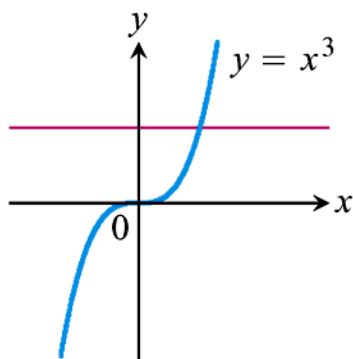
- A function is a rule that assigns a single value in its range to each point in its domain.
- Some functions assign the same output to more than one input.
- Other functions never output a given value more than once.
- If each output value of a function is associated with exactly one input value, the function is *one-to-one*.

One-to-One Functions

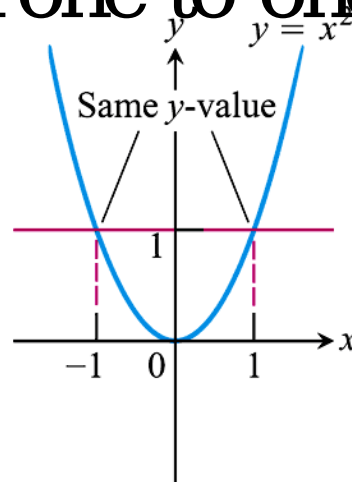
A function $f(x)$ is **one-to-one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.

One-to-One Functions

The horizontal line test states that the graph of a one-to-one function $y = f(x)$ can intersect any horizontal line at most once. If it intersects such a line more than once it assumes the same y -value more than once and is not a one-to-one function.



One-to-one: Graph meets each horizontal line once.



Not one-to-one: Graph meets some horizontal lines more than once.

Inverses

- Since each output of a one-to-one function comes from just one input, a one-to-one function can be reversed to send outputs back to the inputs from which they came.
- The function defined by reversing a one-to-one function f is the **inverse of f** .
- Composing a function with its inverse in either order sends each output back to the input from which it came.

Inverses

The symbol for the inverse of f is f^{-1} , read “ f inverse.” The -1 in f^{-1} is not an exponent;

$f^{-1}(x)$ does not mean $\frac{1}{f(x)}$.

If $(f \circ g)(x) = (g \circ f)(x)$, then f and g are inverses of one another; otherwise they are not.

Identity Function

The result of composing a function and its inverse in either order is the **identity function**.

Example Inverses

Determine via composition if $f(x) = \sqrt{x}$ and $g(x) = x^2, x \geq 0$ are inverses.

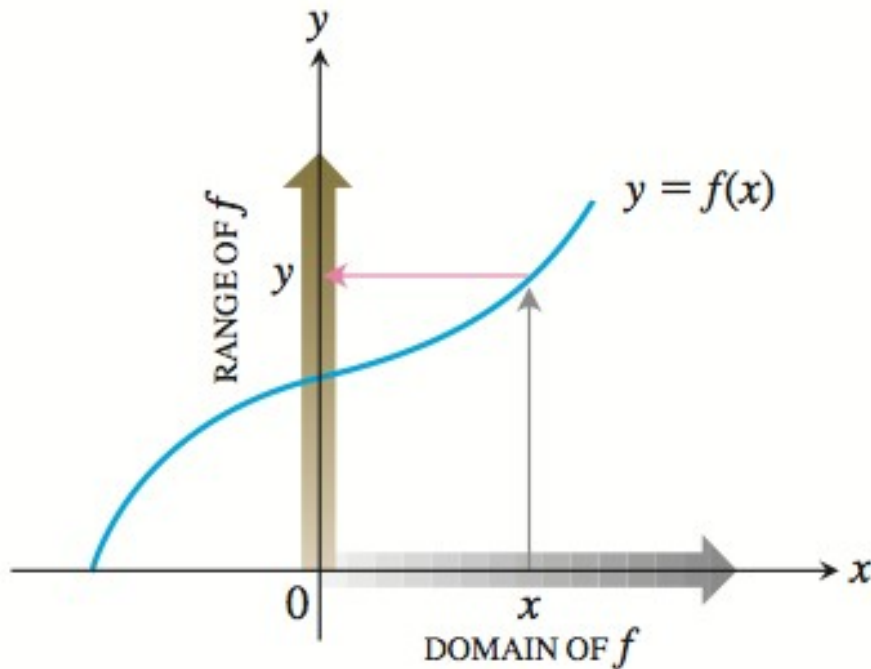
$$(f \circ g)(x) = f(x^2) = \sqrt{x^2} = |x| = x$$

$$(g \circ f)(x) = g(\sqrt{x}) = (\sqrt{x})^2 = x$$

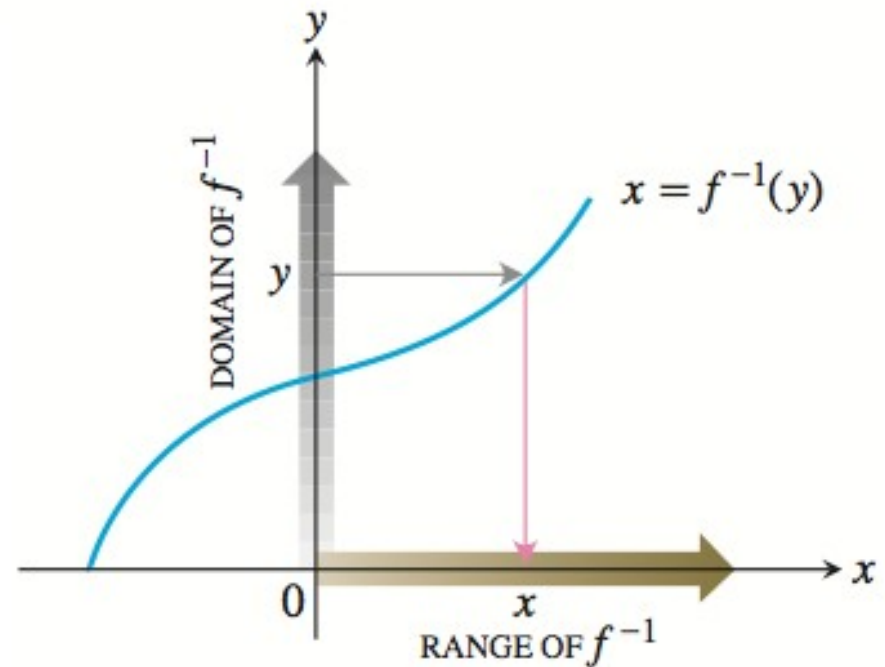
Writing f^{-1} as a Function of x .

1. Solve the equation $y = f(x)$ for x in terms of y .
2. Interchange x and y . The resulting formula will be $y = f^{-1}(x)$.

Finding Inverses

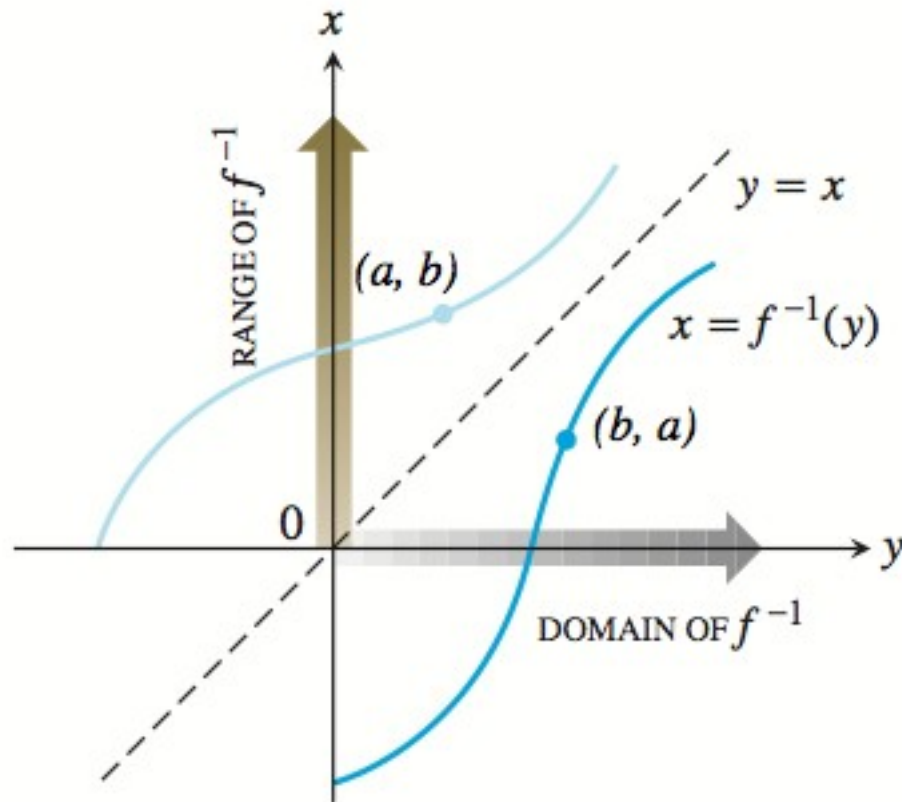


(a) To find the value of f at x , we start at x , go up to the curve, and then over to the y -axis.

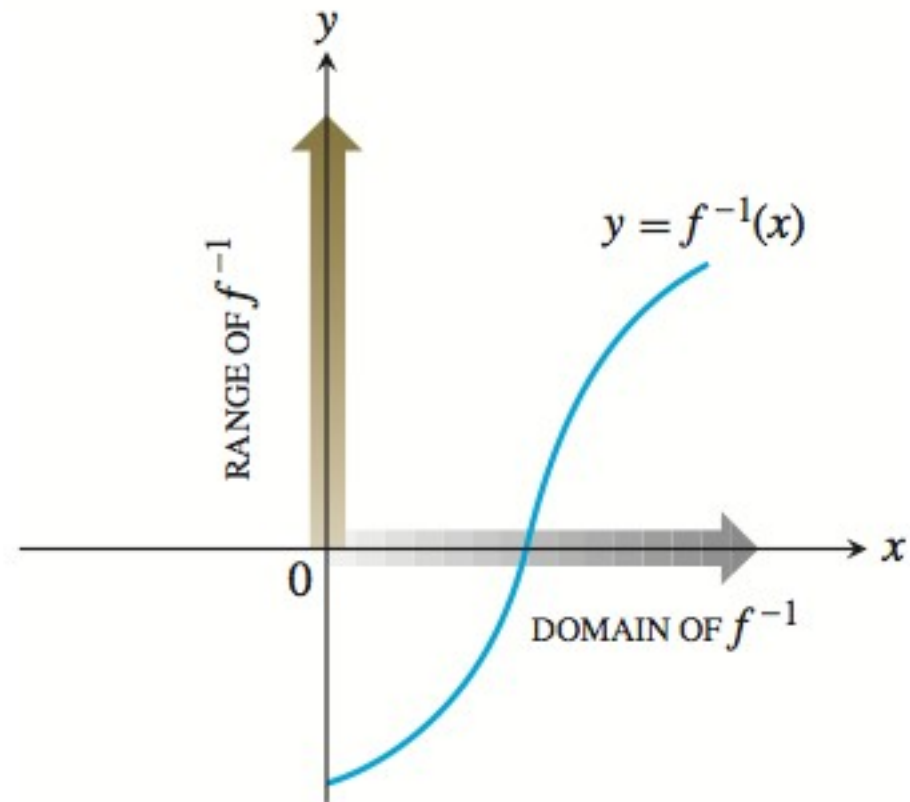


(b) The graph of f is also the graph of f^{-1} . To find the x that gave y , we start at y and go over to the curve and down to the x -axis. The domain of f^{-1} is the range of f . The range of f^{-1} is the domain of f .

Finding Inverses



(c) To draw the graph of f^{-1} in the usual way, we reflect the system across the line $y = x$.



(d) Then we interchange the letters x and y . We now have a normal-looking graph of f^{-1} as a function of x .

Example Finding Inverses

Given that $y=4x-12$ is one-to-one, find its inverse.

Graph the function and its inverse.

$[-10,10]$ by $[-15, 8]$

Solve the equation for x in terms of y .

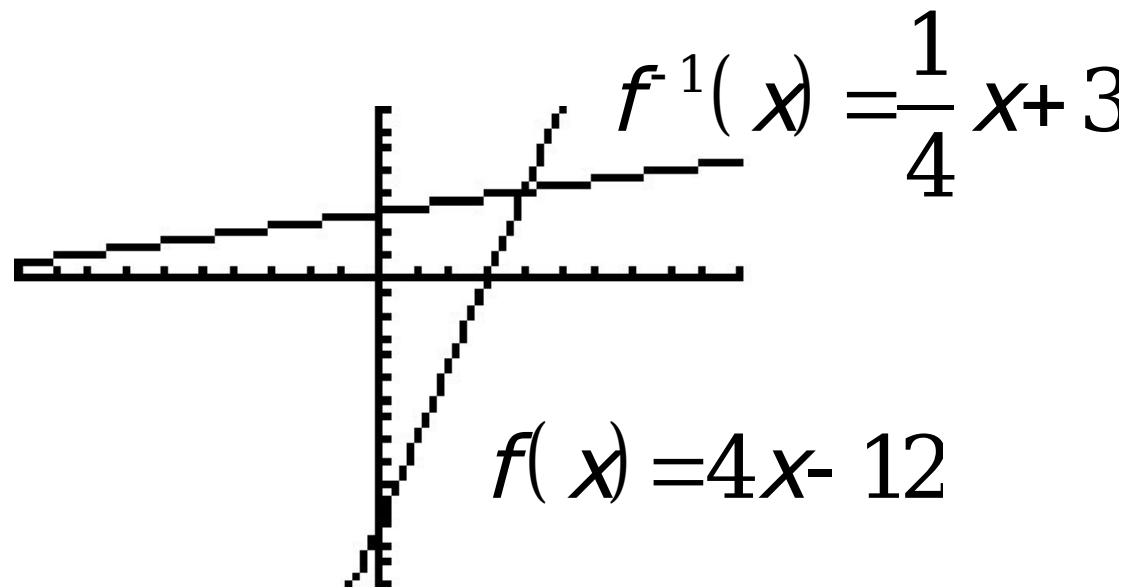
$$x = \frac{1}{4}y + 3$$

Interchange x and y .

$$y = \frac{1}{4}x + 3$$

Notice the symmetry

about the line $y=x$.



$[-10,10]$ by $[-15, 8]$

Base a Logarithmic Function

The base a logarithm function $y = \log_a x$ is the inverse of the base a exponential function $y = a^x$ ($a > 0, a \neq 1$).

The domain of $\log_a x$ is $(0, \infty)$, the range of a^x .
The range of $\log_a x$ is $(-\infty, \infty)$, the domain of a^x .

Logarithmic Functions

- Logarithms with base e and base 10 are so important in applications that calculators have special keys for them.
- They also have their own special notations and names called the
 $y = \log_e x = \ln x$ is called the
natural logarithm function.
 $y = \log_{10} x = \log x$ is often called the
common logarithm function.

Inverse Properties for a^x and $\log_a x$

Base $a > 0$ and $a \neq 1$:

$$a^{\log_a x} = x \text{ for } x > 0; \quad \log_a a^x = x \text{ for all } x$$

Base e :

$$e^{\ln x} = x \text{ for } x > 0; \quad \ln e^x = x \text{ for all } x$$

Properties of Logarithms

For any real numbers $x > 0$ and $y > 0$,

Product Rule: $\log_a xy = \log_a x + \log_a y$

Quotient Rule: $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power Rule: $\log_a x^y = y \log_a x$

Example Properties of Logarithms

Solve the following for x .

$$2^x = 12$$

$$2^x = 12$$

$$\ln 2^x = \ln 12$$

Take logarithms of both sides

$$x \ln 2 = \ln 12$$

Power Rule

$$x = \frac{\ln 12}{\ln 2} = \frac{2.302585}{.693147} \approx 3.32193$$

Example Properties of Logarithms

Solve the following for x .

$$e^x + 5 = 60$$

$$e^x + 5 = 60$$

$$e^x = 55$$

Subtract 5

$$\ln e^x = \ln 55$$

Take logarithm of both sides

$$x = \ln 55 \approx 4.007333$$

Change of Base Formula

$$\log_a x = \frac{\ln x}{\ln a}$$

This formula allows us to evaluate $\log_a x$ for any base $a > 0$, $a \neq 1$, and to obtain its graph using the natural logarithm function on our grapher.

Example Finding Time

The population P of a city is given by

$P = 105,300e^{0.015t}$ where $t=0$ represents 1990.

According to this model, when will the population reach 150,000?

Example Finding Time - Solution

$$1000(1.0525)^t = 2500$$

$$(1.0525)^t = 2.5$$

Divide by 1000

$$\ln(1.0525)^t = \ln 2.5$$

Take logarithm of both sides

$$t \ln 1.0525 = \ln 2.5$$

Power Rule

$$t = \frac{\ln 2.5}{\ln 1.0525} \approx 17.9 \text{ years}$$

The amount in Sarah's account will be \$2500 in about 17.9 years, or about

17 years, 11 months.